Math 1502, **Practice Problem Set for Exam 1**

1. Given approximations for the values of a function,

$$
\begin{aligned}
f(0) &\approx 0.41, \\
f(1) &\approx 0.24, \\
f(2) &\approx 0.15, \\
f(3) &\approx 0.13, \\
f(4) &\approx 0.16, \\
f(5) &\approx 0.22,
\end{aligned}
$$

   integrate $f(x)$ numerically on the interval $[0, 4]$ using $n = 2$ subintervals by

   (a) trapezoid method

   (b) Simpson's method

2. Find the general solution of the linear differential equation

   (a) $y'' - 2y' - 3y = 0$

   (b) $y'' + 8y' + 16y = 0$;

   (c) $8y'' + 2y' - y = 0$.

3. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference of its temperature and the temperature of its environment. Suppose there is a cup of tea sitting on the kitchen table. We measure its temperature to be 210 degrees Fahrenheit. We leave the tea for one minute and then measure its temperature again, getting a second temperature reading of 195 degrees Fahrenheit.

   (a) Assuming the air temperature in the kitchen is 70 degrees Fahrenheit, write down a differential equation for the temperature $T(t)$ of the tea after t minutes.

   (b) Solve the differential equation and find the formula for $T(t)$.

4. Determine if the integral converges and, if so, calculate the integral whenever possible.

   (a)

$$\int_0^1 \frac{1}{(1-t)^2}\, dt$$

   (b)

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\, dx$$

   (c)

$$\int_0^9 \frac{1}{(x-1)^{2/3}}\, dx$$

(d)

$$\int_{10}^{\infty} \frac{x^2+1}{x^3-2}\,dx$$

5. Does the series converge? Does it converge absolutely? Justify your answers.

(a)

$$\sum_{k=0}^{\infty} \frac{k^2+3}{3^k}$$

(b)

$$\sum_{k=1}^{\infty} (-1)^k \frac{\ln k}{\sqrt{k}}$$

(c)

$$\sum_{n=1}^{\infty} \frac{1}{(\ln n)^n}$$

(d)

$$\sum_{k=1}^{\infty} \frac{2k+1}{\sqrt{k^5+1}}$$

(e)

$$\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

(f)

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\sqrt{k(k+1)}}$$

6. Give an upper bound for the error when the partial sum

$$\sum_{k=1}^{24} \frac{1}{\sqrt{k+1}}$$

is used to estimate the sum of the infinite series

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+1}}.$$

7. Expand $g(x) = x^3 \sin(2x^2)$ as a power series in x.

8. Find the Taylor series for e^{-x} centered around $x = 1$, and prove that this series converges to e^{-x} for all real x.

9. Use the Lagrange form of the remainder for Taylor series to estimate $\sqrt{82}$ to within 3 decimal places.

10. Find the radius of convergence for the power series

$$\sum_{k=1}^{\infty} \frac{2^k}{k^2} x^k.$$