Math 1502, **Practice Problem Set for Exam 2**

1. Determine whether the given sets of vectors are linearly dependent/independent.

   (a) $\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix} \right\}$

2. For every system of equations given below perform the following steps:

   - Write the augmented matrix of the system.
   - Reduce the system to reduced echelon form.
   - Find the solutions of the system.
   - Express the solution in vector form.

   (a) $\begin{aligned} 2x + 2y &= 5 \\ x - 4y &= 0 \end{aligned}$

   (b) $\begin{aligned} -x + y &= 1 \\ x + y &= 2 \end{aligned}$

   (c) $\begin{aligned} x - 3y + z &= 1 \\ x + y + 2z &= 14 \end{aligned}$

   (d) $\begin{aligned} -x - y &= 1 \\ -3x - 3y &= 2 \end{aligned}$

   (e) $\begin{aligned} 4y + z &= 20 \\ 2x - 2y + z &= 0 \\ x + z &= 5 \\ x + y - z &= 10 \end{aligned}$

   (f) $\begin{aligned} 2x + z + w &= 5 \\ y - w &= -1 \\ 3x - z - w &= 0 \\ 4x + y + 2z + w &= 9 \end{aligned}$

3. Small apples sell for \$3/pound, while large apples sell for \$5/pound. If 100 pounds of apples were sold for \$360, how many pounds of small/large apples were sold?

4. Describe the set of vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ such that the set

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ x_3 \end{bmatrix}, \begin{bmatrix} 2 \\ x_2 \\ 1 \end{bmatrix}, \begin{bmatrix} x_1 \\ 1 \\ 2 \end{bmatrix} \right\}$$

is linearly dependent.

5. True or false?

   (a) The span of the columns of a matrix A is equal to the image of the linear transformation T given by $T(x) = Ax$.

   (b) Any 3 vectors in $\mathbb{R}^2$ are linearly dependent.

   (c) The span of any 3 vectors in $\mathbb{R}^2$ is $\mathbb{R}^2$.

   (d) The system of equations $Ax = 0$ can be inconsistent.

   (e) Any collection of vectors from $\mathbb{R}^3$ that includes $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$ is linearly dependent.

   (f) The domain of the linear transformation $T(x) = Ax$ with $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$ is $\mathbb{R}^2$.

   (g) The range of the linear transformation $T(x) = Ax$ with $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$ is $\mathbb{R}^2$.

   (h) If the matrix A has more columns than rows then the system $A\boldsymbol{x} = 0$ always has infinitely many solutions.

   (i) If the matrix A has more columns than rows then the system $A\boldsymbol{x} = \boldsymbol{b}$ always has infinitely many solutions, for any vector $\boldsymbol{b}$.

6. Find the standard matrix of the linear transformation T given the information below:

   (a) $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

   (b) $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} z \\ y \\ -z \end{bmatrix}$

   (c) $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 2x + 2y + 3z \\ y + 4z \\ -2z \end{bmatrix}$

7. Consider a linear map $T(\boldsymbol{x}) = A\boldsymbol{x}$, where

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

(a) Find the preimage of $\boldsymbol{b} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$, i.e., find $\boldsymbol{x}$ such that $T(\boldsymbol{x}) = \boldsymbol{b}$.

(b) Find a such that the vector $\begin{bmatrix} a \\ 1 \\ a \end{bmatrix}$ is in the range of T.

(c) Is T onto?

(d) Is T one-to-one?

8. (a) Construct a linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(\boldsymbol{e}_1) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $T(\boldsymbol{e}_2) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, and

- ... is not onto.
- ... is not one-to-one.

(b) Is your "not onto" example one-to-one?

(c) Is your "not one-to-one" example onto?