1. Definitions and statements of results:
   (a) Define what it means for a set S of vectors in a linear space V to *span* V.
   (b) State the Cauchy-Schwartz inequality for vectors x, y in a Euclidean space V, including the condition stating when equality holds.

2. Let $x_1, \ldots, x_n$ be positive real numbers. Define the *arithmetic mean* of $x_1, \ldots, x_n$ to be
$$AM(x_1, \ldots, x_n) = \frac{x_1 + \cdots + x_n}{n},$$
and define the *harmonic mean* of $x_1, \ldots, x_n$ to be
$$HM(x_1, \ldots, x_n) = \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}.$$
Use the Cauchy-Schwarz inequality to show that $HM(x_1, \ldots, x_n) \leq AM(x_1, \ldots, x_n)$, with equality if and only if $x_1 = \cdots = x_n$.

3. Find a unit vector u orthogonal to $v = (1, -3, 2)$ and parallel to the plane $x + 5y - z = 1$. Find the volume of the parallelipiped based on u, v, and the unit normal vector of the given plane.

4. Use Cramer's rule to solve
$$\begin{cases} x + 2y + 3z &= 5 \\ 2x - y + 4z &= 11 \\ -y + z &= 3 \end{cases}$$

5. Determine whether or not S is a subspace of $\mathbb{R}^3$ in the following examples:
   (a) $S = \{(x, y, z) \in \mathbb{R}^3 \;:\; x = y = z\}$
   (b) $S = \{(x, y, z) \in \mathbb{R}^3 \;:\; y = 2x \text{ and } z = 3x\}$

6. Let P_n denote the linear space of all real polynomials of degree at most n. Determine whether or not S is a subspace of P_n in the following examples:
   (a) $S = \{f \in P_n \;:\; f(1) = 0\}$
   (b) $S = \{f \in P_n \;:\; f(x) \geq 0 \text{ for all } x\}$

7. Let P_2 be the linear space of all polynomials of degree at most 2.
   (a) Is $t^2 - 1$ in the subspace of P_2 spanned by $\{1 + t, 1 - t, t^2 + 2\}$?
   (b) Show that the polynomials $f_1 = 1, f_2 = t - 1, f_3 = (t - 1)^2$ form a basis for P_2.
   (c) Find the coordinates of $2t^2 - 5t + 6$ relative to the ordered basis (f_1, f_2, f_3) for P_2.

8. Let $V = C(0,1)$ be the linear space of all continuous functions on the interval $[0,1]$, and define an inner product on V by setting

$$\langle f, g \rangle = \int_0^1 t^2 f(t)g(t)dx.$$

With respect to this inner product:

(a) What are the norms of the functions $f(x) = \pi$ and $g(x) = x$?

(b) What is the cosine of the angle between the functions $f(x) = \pi$ and $g(x) = x$?

9. Let V be the subspace of $\mathbb{R}^3$ spanned by the vectors $x_1 = (1,0,1)$ and $x_2 = (-1,1,2)$.

(a) Find the coordinates (= components) of the vector $v = (0,1,3)$ with respect to the ordered basis (x_1, x_2) for V.

(b) Use the Gram-Schmidt procedure to find an orthogonal basis for V.

10. Let V be a finite-dimensional Euclidean space, let W be a subspace of V, and let $W^\perp$ be the orthogonal complement of W.

(a) Prove that every vector $x \in V$ can be written *uniquely* as $x = y + z$ with $y \in W$ and $z \in W^\perp$.

(b) Prove that $\|x\|^2 = \|y\|^2 + \|z\|^2$.

(c) Prove that $\dim(W) + \dim(W^\perp) = \dim(V)$. [**Hint:** If $Y = \{y_1, \ldots, y_k\}$ is an orthogonal basis for W and $Z = \{z_1, \ldots, z_l\}$ is an orthogonal basis for $W^\perp$, show that $Y \cup Z = \{y_1, \ldots, y_k, z_1, \ldots, z_l\}$ is an orthogonal basis for V.]