1. Definitions and statements of results:
   (a) Describe Gauss-Jordan process of finding a row echelon form of a matrix.
   (b) What are elementary row operations? How can one describe these operations via multiplication by a square matrix on the left?
   (c) What are the axioms that a determinant function should satisfy?

2. Let V and W be linear spaces.
   (a) Define what it means for a function $T : V \to W$ to be a *linear transformation*.
   (b) Define the *image* (= range) of a linear transformation $T : V \to W$.
   (c) Let $T : V \to W$ be a linear transformation, let $\ker(T)$ be the kernel (= null space) of T, and let $\mathrm{im}(T)$ be the image (= range) of T. What does the rank-nullity theorem say about the relationship between the dimensions of the linear spaces $T(V), N(T), V$, and W?
   (d) Define what it means for T to be *onto* and *one-to-one*.

3. Let A be the 3×3 matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ -3 & 1 & 0 \end{pmatrix}$$

   (a) Find a basis for the row space of A.
   (b) Find a basis for the null space of A.

4. Find the reduced row echelon form and the rank of the following matrix:

$$A = \begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 2 & -3 & 1 \\ 2 & 1 & 0 & 3 \end{pmatrix}$$

5. Consider the system of linear equations

$$\begin{aligned} x - 2y + az &= 2 \\ x + y + z &= 0 \\ 3y + z &= 2 \end{aligned}$$

   (a) For which values of a, if any, does this system have a unique solution?
   (b) For which values of a, if any, does this system have no solution?
   (c) For which values of a, if any, does this system have infinitely many solutions?

6. Find the dimension and a basis for the subspace of $\mathbb{R}^5$ consisting of all solutions to the following system of homogeneous linear equations:

$$\begin{aligned} x + 2y - 4z + 3u - v &= 0 \\ x + 2y - 2z + 2u + v &= 0 \\ 2x + 4y - 2z + 3u + 4v &= 0 \end{aligned}$$

7. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is defined as follows: Each vector (x, y) is reflected across the line $y = x$ and then tripled in length to yield $T(x, y)$.

   (a) Determine the matrix A of T with respect to the ordered basis $v_1 = (1, 1), v_2 = (-1, 1)$ for $\mathbb{R}^2$.

   (b) Determine the matrix B of T with respect to the standard basis $e_1 = (1, 0), e_2 = (0, 1)$ for $\mathbb{R}^2$.

8. Let

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}.$$

   (a) Find the inverse of A.

   (b) Find the determinant

      (a) via Gauss-Jordan;

      (b) via cofactor expansion.

   (c) Solve $AX = (3, 3, 14)^T$.

9. For

$$F(x) = \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix}$$

   where $f_{ij}(x)$ are differentiable functions, prove that

$$F'(x) = \begin{vmatrix} f'_{11}(x) & f'_{12}(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f'_{21}(x) & f'_{22}(x) \end{vmatrix}$$

10. Find the least squares approximate solution of the overdetermined system

$$\begin{aligned} x &= 1 \\ 2x - y &= 3 \\ 3x + 5y &= 7 \end{aligned}$$