Math 1512, **Final Exam Review (Part 2)**

Tue, Dec 14th, 2:50pm-5:40pm, Skiles 271. Office hours: Mon 3-5pm.

Topics/keywords:

- Eigenvalues and eigenvectors: computation in the finite-dimensional case, characteristic polynomial.
- Cayley-Hamilton theorem, Jordan canonical form.
- Symmetric/Hermitian maps/matrices: orthonormal basis of eigenvectors (finite-dimensional case), adjoint matrix, unitary/orthogonal matrices.
- Quadratic forms: reduction of real quadratic form to a diagonal form.
- Series: convergence tests, comparison theorems, alternating series.
- Taylor polynomials and series: radius of convergence of power series, estimating approximation error using Lagrange's remainder formula.

Be prepared to state theorems, outline algorithms, and define any of the concepts above.

The following are practice questions:

1. Define what it means for a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ to be *diagonalizable*.
2. Define what it means for a sequence $\{a_n\}$ of real numbers to *converge*.
3. Let
$$A = \begin{pmatrix} 0 & 9 \\ 1 & 0 \end{pmatrix}.$$
   (a) Find all eigenvalues and eigenvectors for A.
   (b) Find a nonsingular matrix C and a diagonal matrix D such that $C^{-1}AC = D$.
4. Find a nonsingular matrix C and a diagonal matrix D for which $D = C^{-1}AC$, where
$$A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$$
5. If T is the linear transformation whose matrix with respect to the standard ordered basis of $\mathbb{R}^2$ is
$$\begin{pmatrix} 1 & 7 \\ 2 & 3 \end{pmatrix},$$
find the matrix of T with respect to the ordered basis $b_1 = (1, -1), b_2 = (2, 1)$ of $\mathbb{R}^2$.
6. Find the eigenvectors, generalized eigenvectore, and the Jordan form of the matrix
$$A = \begin{pmatrix} 2 & 1 & 0 \\ -1 & 4 & 0 \\ -1 & 3 & 1 \end{pmatrix}$$

7. p.235 #15.

8. Does the series
$$\sum_{k=1}^{\infty} \frac{2k+1}{\sqrt{k^5+1}}$$
converge or diverge? Justify your answer.

9. Does the series
$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\sqrt{k(k+1)}}$$
converge absolutely, converge conditionally, or diverge? Justify your answer.

10. Give an upper bound for the error when the partial sum
$$\sum_{k=1}^{24} \frac{1}{\sqrt{k+1}}$$
is used to estimate the sum of the infinite series
$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k+1}}.$$

11. Expand $g(x) = x^3 \sin(2x^2)$ as a power series in x.

12. Find the Taylor series for e^{-x} centered around $x = 1$, and prove that this series converges to e^{-x} for all real x.

13. Use the Lagrange form of the remainder for Taylor series to estimate $\sqrt{83}$ to within 3 decimal places.

14. Find the radius of convergence for the power series
$$\sum_{k=1}^{\infty} \frac{2^k}{k^2} x^k.$$