Homework for the lecture on Dec 2

1. (a) Find the Taylor polynomial $p_2(x)$ for the function $f(x) = \sin(2x)$ centered around $x = 0$.

   (b) Use the Lagrange form of the remainder $r_2(x)$ to prove that $|\sin(1) - 1| < 0.2$.

2. Use Taylor polynomials and Lagrange's remainder formula to estimate $\sqrt{10}$ to within 1 decimal place. (**Hint:** Do not center the Taylor series at $x = 0$.)

3. Derive the Taylor series for the function

$$\ln\left(\frac{1+x}{1-x}\right)$$

   centered at $x = 0$. What is its radius of convergence?

4. The power series

$$\sum_{k=0}^{\infty} a_k x^k$$

   is such that $a_k = a_{k+4}$ for all $k \geq 0$.

   (a) What is its radius of convergence?

   (b) What is the value of the series (assuming it converges)?