1. (a) What is the difference between local and global minima/maxima?
   (b) Summarize Lagrange's method for finding global optima.
   (c) Define the Jacobian matrix of the transformation $F(x,y) = (f_1(x,y), f_2(x,y))^T$.
   (d) Outline Newton's method for approximating solutions of $F(x,y) = 0$.

2. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 10 & -6 \\ -6 & 10 \end{pmatrix}$$

   (b) Write down an orthogonal matrix U such that $U^T AU$ is diagonal.
   (c) Find the singular value decomposition, $A = VDU^T$ for the matrix

$$A = \begin{pmatrix} 3 & -1 \\ 0 & 0 \\ 1 & -3 \end{pmatrix}$$

3. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$$

   (b) Write down an orthogonal matrix U such that $U^T AU$ is diagonal.
   (c) Perform the first step of Jacobi's diagonalization algorithm for the matrix

$$A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 3 \end{pmatrix}$$

4. Set up Newton's method for solving the system of equations: $x^2 + y^2 - 4 = 0$ and $xy - 1 = 0$. Use as an initial guess the point $\boldsymbol{x}_0 = (2,1)^T$ and calculate the next approximation $\boldsymbol{x}_1$.

5. Let $f(x,y) = x^4 + y^4 + 4xy$.
   (a) Use Lagrange's method to find the maximum and the minimum of $f(x,y)$ subject to constraint $g(x,y) = x^2 + y^2 = 16$.
   (b) What are the global maxima and minima of $f(x,y)$ in the region $g(x,y) \leq 16$.

6. Let A be an $m \times n$ matrix of rank r
   (a) Define a singular value decomposition of $A = VDU^T$.
   (b) Define a generalized inverse of A.
   (c) Define the rank s approximation.
7. (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 10 & -6 \\ -6 & 10 \end{pmatrix}$$

   (b) Write down an orthogonal matrix U such that $U^T AU$ is diagonal.
   (c) Find the singular value decomposition, $A = VDU^T$ for the matrix

$$A = \begin{pmatrix} 3 & -1 \\ 0 & 0 \\ 1 & -3 \end{pmatrix}$$

8. For the matrix

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \\ 1 & 2 \end{pmatrix}$$

   (a) Find the singular value decomposition $A = VDU^T$.
   (b) Find a rank 1 approximation of A.
   (c) Find the generalized inverse A^+.
   (d) Using A^+ find the least squares solution $\boldsymbol{x} = (x, y)^T$ for $A\boldsymbol{x} = (1, 1, 1)^T$.