Math 2605, **Final Exam Review (Part 1)**

Thur Dec 16th, 8:00am-10:50am, Skiles 202

Topics/keywords:

- Lines and planes: point-to-plane, point-to-line, line-to-line distances.
- Gradient, tangent lines/planes to implicitly/explicitly given curves/surfaces.
- Newton's method: Jacobian matrix.
- Finding local and global maxima and minima: Hessian, Lagrange's method.
- SVD: generalized inverse, least squares.

Be prepared to state theorems, outline algorithms, and define any of the concepts above.

1. Find the distance between the tangent line to the curve $\boldsymbol{x}(s) = (s^2-1, s^3-s+2, 2s-1)^T$ at the point $\boldsymbol{x}_0 = (0, 2, 1)$ and the line $\boldsymbol{x}(t) = (-a-1-t, a+t, a+t)^T$.

2. The temperature T in a metal ball is inversely proportional to the distance from the center of the ball, which we take to be the origin. The temperature at the point $\boldsymbol{x}_0 = (1, 2, 2)$ is 120 degrees.
   (a) Find the rate of change of T at $\boldsymbol{x}_0$ in the direction toward the point $(2, 1, 3)$.
   (b) Show that at any point in the ball the direction of greatest increase in temperature is given by a vector that points at the origin.

3. Consider the graph of $f(x, y) = (x-1)^2 + (y-2)^2 + 3$. Write an equation for the tangent plane at the point $\boldsymbol{x}_0 = (3, 2, 7)$. At what point is the tangent plane horizontal?

4. Suppose a circular metal wire $x^2 + y^2 = 1$ is heated so that the temperature of the wire at (x, y) is $T(x, y) = x^2 + 2y^2 - x$. Where are the hottest and coldest points on the wire?

5. Find the critical points of $f(x, y) = x \sin y$ and determine whether they are local minima, local maxima or saddle points.

6. Set up Newton's method for solving the system of equations: $2xy + y^2 + 4y = 0$ and $xy + 2y - 1 = 0$. Use as an initial guess the point $\boldsymbol{x}_0 = (1, -1)^T$ and calculate the next approximation $\boldsymbol{x}_1$.

7. Consider

$$A = \begin{bmatrix} 1 & 1 \\ 1-a & 1 \\ 1+a & 1 \end{bmatrix}$$

   (a) Compute a singular value decomposition of A.
   (b) Compute $A_{(1)}$ the best rank 1 approximation of A. What is the norm of the difference $A - A_{(1)}$?
   (c) Compute the least length, least squares solutions to both $A\boldsymbol{x} = \boldsymbol{b}$ and $A_{(1)}\boldsymbol{x} = \boldsymbol{b}$ where $\boldsymbol{b} = (1, 3, 2)^T$.