Thur Dec 16th, 8:00am-10:50am, Skiles 202. Office hours: Wed 3-5pm.

Topics/keywords:

- Schur factorization: 2×2 case.
- Householder reflections: real and complex cases.
- QR iteration: algorithm, cases when it is guaranteed to work, shifting eigenvalues.
- Matrix exponentials: exponentiating upper triangular matrices, using Schur factorization.
- Double integrals: type I and II repeated integrals, integrals in polar coordinates.
- Applications of integrals: area, volume.
- Change of variables in integrals: Jacobian matrix, linear coordinate transformations, general transformations.

Be prepared to state theorems, outline algorithms, and define any of the concepts above.

1. Establish a Schur decomposition $A = QTQ^T$ with an upper-triangular T and orthogonal Q for
$$A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}.$$

2. Find a Householder reflection M mapping $\boldsymbol{x} = (3i, 4)^T$ onto a multiple of $\boldsymbol{e}_1$.

3. Given a matrix A, describe how the sequence $A^{(k)}$ is created via QR iteration.

   Consider the matrix
$$A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}.$$
   Compute its QR decomposition using a "good" Householder reflection. Compute the matrix $A^{(1)}$ obtained after one QR iteration.

4. Prove that an eigenvector of A is an eigenvector of $A + aI$ for any a. What is the "eigenvalue shifting" procedure?

5. Prove that if $(A - aI)$ is an invertible matrix then $(A - aI)^{-1}$ has the same eigenvalues as A. How is this fact used to speed up the QR iterations if a rough approximation a of an eigenvalue μ is known?

6. Give a definition of e^A for a square matrix A. Given a Schur decomposition $A = UTU^*$ rewrite e^A in terms of e^T.

7. Evaluate
$$\iint_\Omega \frac{2y}{x^2+1}\, dx\, dy,$$
   where Ω is the region bounded by $x = 0$, $x = 1$, $y = 0$, $y^2 - x = 0$.

8. Consider the solid bounded below by the xy-plane and above by the surface $z = \cos(x^2 + y^2)$ and containing a part of z-axis.
   (a) Sketch the projection Ω of the solid onto the xy-plane.
   (b) Find the volume of the solid using polar coordinates.

9. The region Ω in the plane is bounded by the lines $x + y = 0$, $x + y = 1$, $3x - 2y = 0$, and $3x - 2y = 2$. Calculate the following integral

$$\iint_\Omega (4x - y)\, dx\, dy.$$

10. Calculate using polar coordinates

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{-(x^2+y^2)}\, dx\, dy.$$

11. Sketch the *petal curve* given by $r = 3\sin(3\theta)$ in the polar coordinates and find the area of *one leaf* of the region bounded by this curve.