1. Given a matrix A, describe how the sequence $A^{(k)}$ is created via QR iteration.

2. Suppose A is a real square matrix with real eigenvalues. Give a criterion for the sequence $A^{(k)}$ to converge to an upper-triangular matrix.

3. Prove that an eigenvector of A is an eigenvector of $A + aI$ for any a. What is the "eigenvalue shifting" procedure?

4. Let Q be a unitary transformation obtained in the k-th step of the Schur factorization algorithm: for a $k \times k$ matrix B it is such that Q^*BQ has a multiple of $\mathbf{e}_1$ in the first column.

   How does one use Q to create a unitary transformation in the n-dimensional space? How is the latter used to update the output unitary matrix U?

   Upon termination the algorithm produces a Schur decomposition $A = UTU^*$, where T can be computed as the following product of matrices: U^*AU.

   How would you modify the algorithm to produce T without computing this product at the end?

5. Prove that if $(A - aI)$ is an invertible matrix then $(A - aI)^{-1}$ has the same eigenvalues as A.

   How is this fact used to speed up the QR iterations if a rough approximation a of an eigenvalue μ is known?

6. CAR: Chapter 4: p.1-41: 1.

7. Give a definition of e^A for a square matrix A. Given a Schur decomposition $A = UTU^*$ rewrite e^A in terms of e^T.

8. CAR: Chapter 4: p.1-50: 2.

9. Given a second order ODE with constant coefficients, what is a way to phrase the problem as a system of two first order ODEs?

10. CAR: Chapter 5: p.1-50: 5.