Name:
Quiz 1
Math 2605
14 Jan 10

ANSWER KEY

$(1,-2,4)^t$

Consider the plane in R^3 containing the points $p_1 = (1,-2,4)^t$, $p_2 = (2,0,1)^t$, and $p_3 = (-1,1,3)^t$. $(2,0,1)^t \longrightarrow$ $\longleftarrow (-1,1,3)^t$

1. Write a parametrization for the plane using a base point x_0 and direction vectors v_1 and v_2. ~~(7 points)~~ (5 pts)

2. Write an equation for the plane using a normal vector, base point, and the dot product. ~~(8 points)~~ (5 pts)

① Let $\vec{x}_0 = \vec{p}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$, $\vec{v}_1 = \vec{p}_2 - \vec{p}_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

$$\vec{v}_2 = \vec{p}_3 - \vec{p}_1 = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$$

then $$\vec{x}(s,t) = \vec{x}_0 + s\vec{v}_1 + t\vec{v}_2 = \begin{bmatrix} 1+s-2t \\ -2+2s+3t \\ 4-3s-t \end{bmatrix}$$

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② normal vector: $\vec{a} = \vec{v}_1 \times \vec{v}_2$

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \times \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -2-(-9) \\ (-3)(-2)-(-1) \\ 3\cdot 1-(-2)(2) \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix}$$

basepoint $= \vec{x}_0 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

$$\vec{a}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 7x+7y+7z$$

equation: $\vec{a}\cdot\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \vec{x}_0\right) = 0 \Rightarrow \vec{a}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \vec{a}\cdot\vec{x}_0$

$$\vec{a}\cdot\vec{x}_0 = \begin{bmatrix} 7 \\ 7 \\ 7 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} = 7-14+28 = 21$$

$\therefore\ 7x+7y+7z = 21$ | $\boxed{x+y+z=3}$