

Math 2605-C Quiz 11  
15 Apr 10

Name: SOLUTIONS

1. (8 points) Compute  $e^{tA}$ , where  $A = \begin{bmatrix} 3 & 5 \\ 0 & 7 \end{bmatrix}$
2. (2 points) Compute  $\frac{d}{dt}e^{tA}$ .

$$\textcircled{1} \quad e^{tA} = \begin{bmatrix} e^{3t} & a \\ 0 & e^{7t} \end{bmatrix}$$

where we need to find  $a$ .

Use that  $Ae^{tA} = e^{tA}A$ , so:

$$Ae^{tA} = \begin{bmatrix} 3e^{3t} & 3a + 5e^{7t} \\ 0 & 7e^{7t} \end{bmatrix}$$

$$= e^{tA}A = \begin{bmatrix} 3e^{3t} & 5e^{3t} + 7a \\ 0 & 7e^{7t} \end{bmatrix}$$

$$\Rightarrow 3a + 5e^{7t} = 5e^{3t} + 7a$$

$$\Rightarrow a = \frac{5}{4}(e^{7t} - e^{3t})$$

$$\Rightarrow e^{tA} = \begin{bmatrix} e^{3t} & \frac{5}{4}(e^{7t} - e^{3t}) \\ 0 & e^{7t} \end{bmatrix}$$

(b) either differentiate  
componentwise or use that

$$\frac{d}{dt} e^{tA} = A e^{tA}$$

either way, get:

$$\frac{d}{dt} e^{tA} = \begin{bmatrix} e^{3t} & \frac{5}{4}(7e^{7t} - 3e^{3t}) \\ 0 & 7e^{7t} \end{bmatrix}$$