

Math2605C Quiz4

Name:

Feb4,2010

Given a function $f(x, y) = x^2y^3$.

1. (3 points) Find an explicit representation of the level set $f(x, y) = c$.

Solution:

$$x^2y^3 = c$$
$$y = \sqrt[3]{\frac{c}{x^2}}$$

2. (3 points) Compute the gradient of $f(x, y)$.

Solution:

$$\nabla f(x, y) = \begin{bmatrix} 2xy^3 \\ 3x^2y^2 \end{bmatrix}$$

3. (4 points) Consider the point $p = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ on the curve $f(x, y) = -1$. Compute the tangent line to the curve at this point.

Solution:

$$\nabla f(-1, -1) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

The equation is

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} \cdot \left(\vec{x} - \begin{bmatrix} -1 \\ -1 \end{bmatrix} \right) = 0$$
$$2x + 3y = -5$$