Math2605-C Quiz6

Name:

Feb25,2010

Let $A = \begin{bmatrix} 2 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$.

1. (7 pt)Compute a singular value decomposition of A.
   Solution:
$$A^t A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$
$$\mu_+ = 9, \mu_- = 1.$$
$$D = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$
   Eigenvectors are $u_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, u_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.
$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$V = AUD^{-1} = \frac{1}{3\sqrt{2}} \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 1 & -3 \end{bmatrix}$$

2. (3 pt)Use the singular value decomposition of A to compute a least squares solution to $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.
   Solution:
$$A^+ b = UD^{-1}V^t = \frac{1}{3} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$