

Math2605-C Quiz8

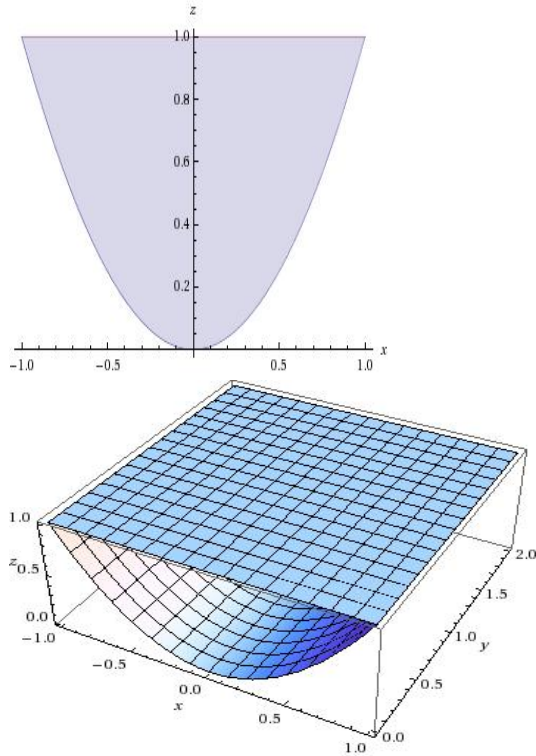
Name:

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T is the solid bounded below by the cylindrical surface $z = x^2$, above by the plane $z = 1$, and on the sides by the planes $y = 0$ and $y = 2$.

1. (4 pt) Sketch T . (Hint: You can first sketch the graph in the xz plane.)

Solution:



$$0 \leq y \leq 2, -1 \leq x \leq 1, x^2 \leq z \leq 1$$

Or

$$0 \leq y \leq 2, 0 \leq z \leq 1, -\sqrt{z} \leq x \leq \sqrt{z}$$

2. (6 pt) Compute the triple integral $\iiint_T 3x^2 y z dx dy dz$.

Solution:

$$\begin{aligned}\iiint_T 3x^2yzdx dy dz &= \int_0^2 \int_{-1}^1 \int_{x^2}^1 3x^2yz dz dx dy \\ &= \int_0^2 \int_{-1}^1 \frac{3}{2}x^2yz^2 \Big|_{x^2}^1 dx dy \\ &= \int_0^2 \int_{-1}^1 \frac{3}{2}(x^2 - x^6)y dx dy \\ &= \int_0^2 \left(\frac{1}{2}x^3 - \frac{3}{14}x^7\right)y \Big|_{-1}^1 dy \\ &= \int_0^2 \frac{4}{7}y dy \\ &= \frac{8}{7}\end{aligned}$$