Math 2605-C Quiz 9 **Name:**

1 Apr 10

Consider the following matrix:

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$$

1. (4 points) Find a normalized eigenvector $\mathbf{u}$ of A and a unit length vector $\mathbf{u}^\perp$ orthogonal to $\mathbf{u}$.
2. (6 points) Find an orthogonal matrix Q and an upper triangular matrix T such that $A = QTQ^{-1}$

By inspection (or by looking at the char poly $(4-\lambda)(5-\lambda)-2$) we see that $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an e-vector with e-value 6.

Hence let $\vec{u} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$,

and $\vec{u}^\perp = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

note: $\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$ is also OK

then let $\boxed{Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}}$

We want $A = QTQ^{-1}$,

so $T = Q^{-1}AQ$

since Q is orthogonal, $Q^{-1} = Q^t$

and $T = Q^t A Q$

$$T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 6/\sqrt{2} & -2/\sqrt{2} \\ 6/\sqrt{2} & 4/\sqrt{2} \end{bmatrix}$$

$$\boxed{T = \begin{bmatrix} 6 & 1 \\ 0 & 3 \end{bmatrix}}$$

← note: since there are 2 choices for λ and u^t, other correct answers exist