Math 2605-M Quiz 9 **Name:**

1 Apr 10

I called it u instead of v below. Let u = v

Consider the complex vector $\mathbf{v} \in \mathbb{C}^2$ given by

$$\mathbf{v} = \begin{bmatrix} 3+2i \\ 5i \end{bmatrix}$$

1. (4 points) Find a vector $\mathbf{w} \in \mathbb{C}^2$ orthogonal to $\mathbf{v}$.
2. (6 points) Find a unitary 2 x 2 matrix whose first column is a real multiple of $\mathbf{v}$.

(1) In general, if $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, we can let $w = \begin{bmatrix} -\bar{z}_2 \\ \bar{z}_1 \end{bmatrix}$. Then $\langle u, w \rangle =$

$= \bar{u} \cdot w = \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \end{bmatrix} \cdot \begin{bmatrix} -\bar{z}_2 \\ \bar{z}_1 \end{bmatrix} = 0,$

So, here let $\boxed{w = \begin{bmatrix} 5i \\ 3-2i \end{bmatrix}}$

(2) $|u| = \sqrt{\langle u, u \rangle} = \sqrt{\bar{u} \cdot u} = \sqrt{9+4+25}$

$= \sqrt{38}$. Similarly $|w| = \sqrt{38}$

Then let $A = \frac{1}{\sqrt{38}} \begin{bmatrix} 3+2i & 5i \\ 5i & 3-2i \end{bmatrix}$

Then $A^* A = I$

and the first column of

A is $\frac{u}{|u|} = \frac{1}{\sqrt{38}} u$,

as desired