Algebraic Geometry: additional exercises (due Sep 7)

1. Let $\phi : \mathbb{A}^n \to \mathbb{A}^n$ be an automorphism of teh affine space $\mathbb{A}^n$ with the coordinate ring

$$k[\mathbb{A}^n] = k[\boldsymbol{x}] = k[x_1, \ldots, x_n].$$

Define the *Jacobian* map

$$J(\phi) : \mathbb{A}^n \to \mathbb{A}^{n \times n}$$

(think: $\mathbb{A}^{n \times n} = \{\text{square matrices}\}$) as

$$J(\phi)_{ij} = \frac{\partial \phi_i}{\partial x_j} \in k[\boldsymbol{x}],\ \ i, j = 1, \ldots n.$$

Show that if ϕ is an isomorphism then

$$\det J(\phi) : \mathbb{A}^n \to \mathbb{A}^1$$

is a nonzero constant map.

2. Given a rational map

$$\rho : V - - \to W$$

consider its *graph*

$$\Gamma_\rho = \overline{\{(\boldsymbol{x}, \rho(\boldsymbol{x}) \mid \boldsymbol{x} \in V \setminus \Sigma(\rho)\}} \subset V \times W.$$

Show that the natural projection (on the first coordinate)

$$\pi : \Gamma_\rho - - \to V$$

(together with ρ) gives a birational equivalence of V and Γ_ρ.