Algebraic Geometry: additional exercises (due Sep 21)

1. Prove that for a given ideal $I \subset k[\boldsymbol{x}]$ the set of all possible initial ideals

$$\{\, \mathrm{in}_{>} I \mid > \text{ is a monomial order on } k[\boldsymbol{x}] \,\}$$

   is finite.

2. A finite set $G \subset k[\boldsymbol{x}]$ is called a *universal Gröbner basis* of I if $\mathrm{in}_{>} I = \langle \mathrm{in}_{>} G \rangle$ for any monomial order $>$. Prove that a universal Gröbner basis exists for every polynomial ideal I.