

## Algebraic Geometry: additional exercises (due Sep 28)

1. If  $f$  and  $g$  are polynomials of the same degree  $d$ ,

$$f = \sum_{k=0}^d a_k x^k, \quad g = \sum_{l=0}^d b_l x^l,$$

then the following method for computing the resultant is often used. Compute the following polynomial in two variables, which is called the *Bézoutian*:

$$B(x, y) = \frac{f(x)g(y) - f(y)g(x)}{x - y} = \sum_{i,j=0}^{d-1} c_{ij} x^i y^j.$$

The entries of the symmetric  $d \times d$ -matrix  $C = (c_{ij})$  are sums of expressions  $a_k b_l - a_l b_k$ .

The *Bézout resultant* is defined as the determinant of  $C$ . Prove that it equals the Sylvester resultant  $Res(f, g)$ .